THREE MODE CONTROLLER (PID) TUNING

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Introduction

What is a control system? Simply put a control system is a means of controlling a process. The process could be anything from level in a tank, pressure in a pipeline, temperature of a fluid or even the temperature in your home.

There are many different methods of controlling the process but by far the most common method used in industrial control systems is the proportional-integral-derivative controller or PID. PID control is widely used due to its simplicity, familiarity, and ease of tuning. In most cases it provides good control, accuracy, and stability.

Control System Theory

In order to control the process at a desired setpoint (SP) the process must be measured. This measurement is called the process variable or PV. To measure the process, a sensor or instrumentation of some kind is used. As an example, if controlling pressure a pressure transmitter (PT) could be used that would send a signal back to the control system.

A control system can be either open-loop or closed-loop. An open-loop control system is one in which the process variable is not compared to the setpoint. Because of this the control system cannot be guaranteed to achieve the desired setpoint. In a closed-loop, or feedback, control system the PV is compared to the SP and the difference is termed the error. A desire for more accurate control is the reason a closed-loop control system is more commonly used.

The means of controlling a process will depend on the process itself but in most cases, there will be a single device that is manipulated to control it. That device is known as the final control element. This device could be a valve, a compressor, a heating element, a pump or any number of devices.

In a closed-loop control system the control system takes in the setpoint and process variable and from that determines what needs to be done to bring the process under control. The output of the control system, called the control variable, or CV, is then sent to the final control element.

When trying to optimize a control system taking a step change in control variable with an open-loop system can provide valuable information about the process. Likewise taking a step change in setpoint on a closed-loop system can provide valuable information about the controller and overall process. In both cases this is called the step response. Below is a typical first-order step response with dead time.

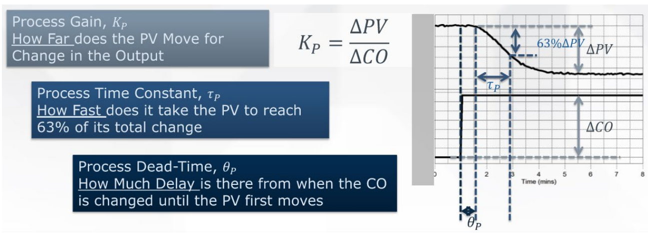


Figure 1
(Rice, 2024)

- Delay or dead time (θ_p)
- Time constant (τ_p), time to reach 63% steady-state value
- Process gain (K_p), change in PV over change in CV

And below is a typical second-order step response with dead time.

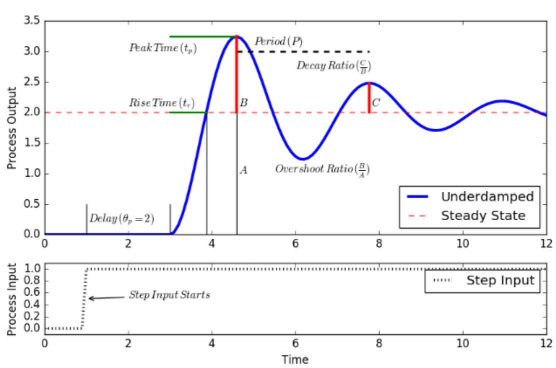


Figure 2
(Hedengren, 2025)

- Delay or dead time (θ_p)
- Rise time (t_r), time from 10% to 90% of steady-state value
- Peak time (t_p), time to reach peak
- Settling time (t_s), time to reach $\pm 2\%$ steady-state value
- Overshoot ratio, ($\frac{B}{A}$, %OS)
- Decay ratio, ($\frac{C}{B}$, % amplitude decay)

PID Control

Below is the basic block diagram for a PID controller in the time domain.

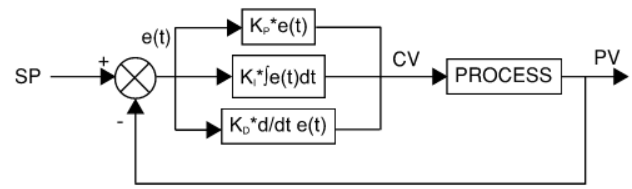


Figure 3

The proportional term is a simple multiplication of the proportional gain and the error term. The integral term is the integral gain times the error integrated with respect to time. The derivative term is, similarly, the derivative gain times the derivative of the error with respect to time. The three terms are summed together resulting in the CV output to the final control element. The characteristics of the final control element and downstream process are simply shown here as "process." The process is often difficult to characterize and, for the purpose of tuning, is often irrelevant as it can be modeled empirically.

In the time domain the equation for the PID algorithm is as follows:

$$G(t) = k_p e(t) + k_i \int e(t)\,dt + k_d \frac{d}{dt} e(t)$$

Using Laplace transform this can be simplified to an algebraic equation in the frequency domain as follows:

$$G(s) = k_p + \frac{k_i}{s} + k_d s$$

If we ignore the contribution of the process (a gain of 1) and calculate the closed-loop transfer function it becomes:

$$\frac{G(s)}{1+G(s)}$$

Then multiply by a unit step function, $\frac{1}{s}$, to get the step response of the closed-loop transfer function.

$$\frac{G(s)}{s+sG(s)}$$

Substituting in G(s) and rewriting it in the standard form gives the following.

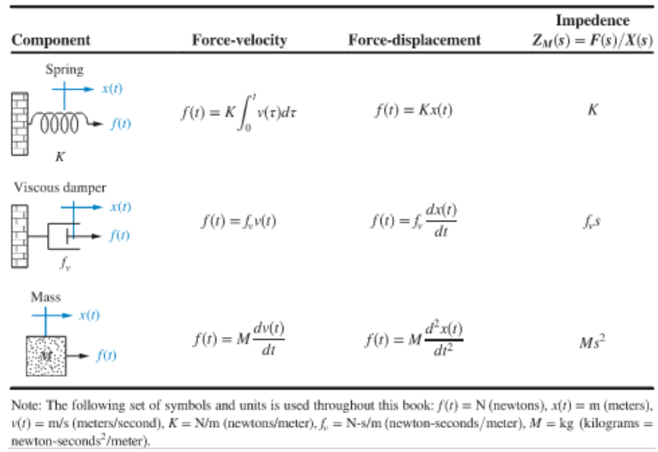
$$\frac{\omega_n^2}{s^2+2\zeta\omega_n S+\omega_n^2}$$

This is the standard form of the second order response where ω_n is the natural frequency and ζ is the damping ratio.

This is not intended to be a crash course on control system theory but rather an introduction to some of the basic terms and equations. If you'd like to learn more about control system theory there are many resources online that can provide more information and hopefully this will give you enough information to seek them out.

Mechanical Analogs

Simple mechanical systems can be analogous to these control system transfer functions and can provide some insight into how changing different aspects of those transfer functions can change the output of these control systems. The three basic elements of the simple mechanical system are a spring, a viscous damper, and mass. These elements are summarized in the table below.



Component	Force-velocity	Force-displacement	Impedence $Z_M(s) = F(s)/X(s)$
Spring ($x(t)$, $f(t)$, K)	$f(t) = K\int_0^t v(\tau)d\tau$	$f(t) = Kx(t)$	K
Viscous damper ($x(t)$, $f(t)$, f_v)	$f(t) = f_v v(t)$	$f(t) = f_v\frac{dx(t)}{dt}$	$f_v s$
Mass ($x(t)$, $f(t)$, M)	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

Note: The following set of symbols and units is used throughout this book: $f(t)$ = N (newtons), $x(t)$ = m (meters), $v(t)$ = m/s (meters/second), K = N/m (newtons/meter), f_v = N-s/m (newton-seconds/meter), M = kg (kilograms = newton-seconds^2/meter).

Table 1

(Nise, 2015)

The force exerted by an ideal spring is directly proportional to its displacement, x. The force exerted by a viscous damper is directly proportional to its velocity or the derivative of displacement, $\frac{dx}{dt}$. The force exerted by a mass is directly proportional to its acceleration (f=ma) or the second derivative of displacement, $\frac{d^2x}{dt^2}$. Below is an example.

Transfer Function—One Equation of Motion

PROBLEM: Find the transfer function, $X(s)/F(s)$, for the system of Figure 2.15(*a*).

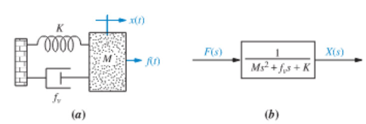


FIGURE 2.15 a. Mass, spring, and damper system; b. block diagram

SOLUTION: Begin the solution by drawing the free-body diagram shown in Figure 2.16(*a*). Place on the mass all forces felt by the mass. We assume the mass is traveling toward the right. Thus, only the applied force points to the right; all other forces impede the motion and act to oppose it. Hence, the spring, viscous damper, and the force due to acceleration point to the left.

We now write the differential equation of motion using Newton's law to sum to zero all of the forces shown on the mass in Figure 2.16(*a*):

$$M\frac{d^2x(t)}{dt^2}+f_v\frac{dx(t)}{dt}+Kx(t)=f(t) \qquad (2.108)$$

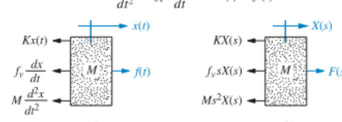


FIGURE 2.16 a. Free-body diagram of mass, spring, and damper system; b. transformed free-body diagram

Taking the Laplace transform, assuming zero initial conditions,

$$Ms^2X(s)+f_vsX(s)+KX(s)=F(s) \qquad (2.109)$$

or

$$(Ms^2+f_vs+K)X(s)=F(s) \qquad (2.110)$$

Solving for the transfer function yields

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + f_v s + K} \qquad (2.111)$$

which is represented in Figure 2.15(b).

Example 1
(Nise, 2015)

From the above example you can see how the resulting transfer function is of the same form as the standard second order transfer function above. As such we can expect a mass attached to a spring and viscous damper to have a similar response as shown in Figure 2.

PID Tuning by Feel

In my experience greater than 90% of PID loops can be adequately tuned by feel and as such we'll start there. Tuning by feel is as much an art as a science but there are a few basic steps I follow when doing it. To start, make sure the process is safe and can be upset as necessary. If you are constrained to certain process parameters make sure those are defined and safeguards are in place to prevent exceeding those process limits. An example of a safeguard could be, if you're tuning a loop on a regulator, you could reduce the upstream pressure to the upper limit of the downstream pressure. If this provides enough of a pressure cut it will prevent you from exceeding your downstream pressure limit.

Once safe to proceed zero out the integral and derivative gain, enter a proportional gain of one, and place the PID in auto. Once in auto make setpoint changes observe the reaction of CV and PV. Continue this process increasing the proportional gain with each step until oscillations are observed in the PV. Reduce the proportional gain until the oscillations disappear then reduce it another ~25%.

From this point the process should be reacting well to setpoint changes but there will be some steady-state error. Increase the integral gain to eliminate the steady-state error. In most cases you won't need derivative action so at this point your loop should be well tuned. Make several setpoint changes or introduce other upsets to test the tuning. If further tuning is required see the chart below to see how different factors affect the tuning.

Closed-Loop Response	Rise Time	Overshoot	Settling Time	Steady-State Error	Stability
Increasing K_P	Decrease	Increase	Small Increase	Decrease	Degrade
Increasing K_I	Small Decrease	Increase	Increase	Large Decrease	Degrade
Increasing K_D	Small Decrease	Decrease	Decrease	Minor Change	Improve

Table 2
(Ang, 2005)

Ziegler-Nichols PID Tuning

The Ziegler-Nichols method of PID tuning is a relatively simple method. As with any tuning method begin by making sure it can be done safely and put safeguards in place as required. Next place the PID in auto with the integral and derivative terms zeroed out, similar to tuning by feel. Begin increasing proportional gain until the process oscillates at a constant magnitude (doesn't decay or increase). The gain at this point is termed the ultimate gain or K_u. The period of this oscillation is T_u. The values for K_u and T_u are used as shown in the table below to derive the proportional gain, K_p, integral gain, K_i, and derivative gain, K_d.

Control Type	K_p	K_i	K_d
P	$0.5K_u$	–	–
PI	$0.45K_u$	$0.54K_u/T_u$	–
PD	$0.80K_u$	–	$0.10K_uT_u$
PID	$0.60K_u$	$1.20K_u/T_u$	$0.075K_uT_u$

Table 3
(Ziegler, 1942)

There are several issues with this tuning method. To begin with you start by taking the process into undamped oscillations. This is often undesirable or unacceptable. Next it provides aggressive tuning with overshoot and quarter-amplitude decay. Again, in many processes this is undesirable.

First-Order Plus Dead Time (FOPDT) Method

This method of tuning involves finding the open-loop step response of the process and fitting that to a first-order plus dead time model. The resulting parameters of that model are then used to calculate PID parameters. This is the method used by auto-tune functions and produces consistent results.

The steps I show in this method were presented by Robert Rice at the Rockwell Automation Fair and consist of six steps but there are other methods that follow the same basic process.

The first step in the process is to find and define the control objectives. You need to find the process variable you're trying to control and the final control element though which that control is achieved. In the example below the control objective is to control the level in the reflux drum. This could be accomplished by controlling the flow into the reflux drum by varying the temperature of the distillation column or by controlling the flow out of the reflux drum through FIC-L. In this case they chose the latter.

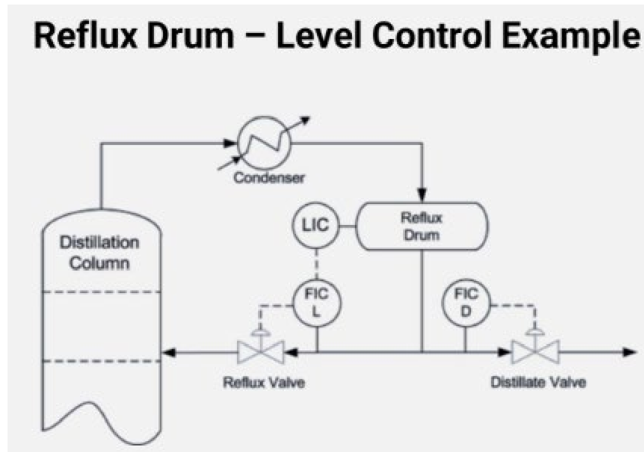


Figure 4
(Rice, 2024)

The second step in the process is to bump the process. This involves placing the PID in manual (open-loop control) and making changes to CV and observing how the process reacts. Changes made to the CV must be sharp and significant. Ideally the change in CV will be of a step form but it's usually not possible to change the final control element instantaneously so it must be done as quickly as possible. To be significant and ensure the final control element reacts the change in CV must be at least 3%.

The third step in the process is to model the process. This involves first identifying the type of process you're dealing with. Below are two types of FOPDT models. Based on the step response identify which form you're dealing with.

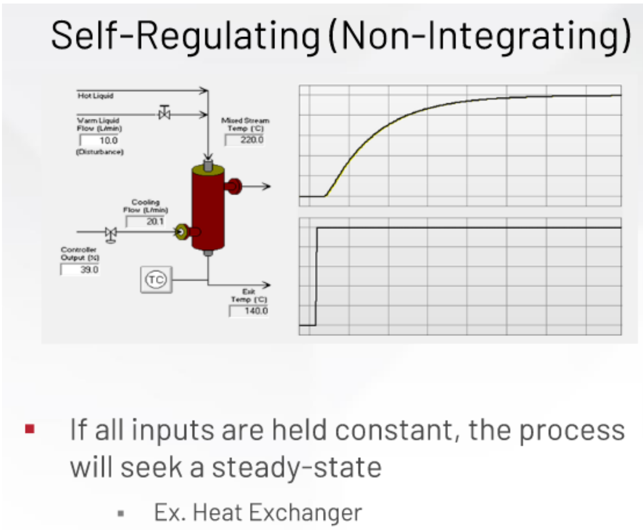


Figure 5
(Rice, 2024)

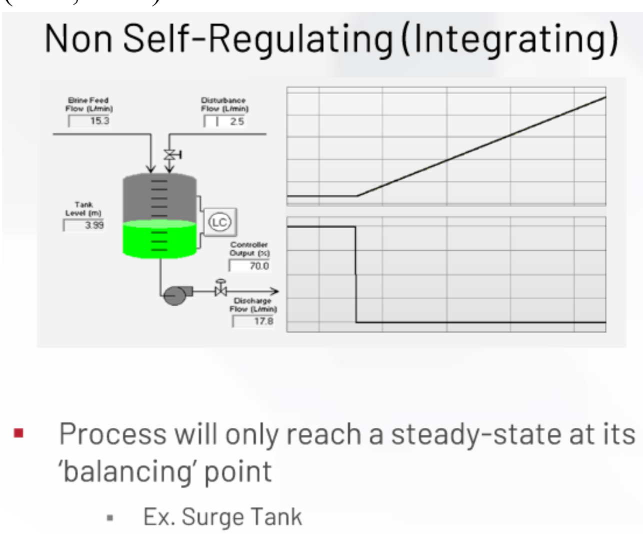


Figure 6
(Rice, 2024)

If the process is self-regulating find the dead time (θ_p), time constant (τ_p), and process gain (K_p) as shown in figure 1. If the process is non-self-regulating find the dead time (θ_p), and process gain (K_p) as shown in the figure below.

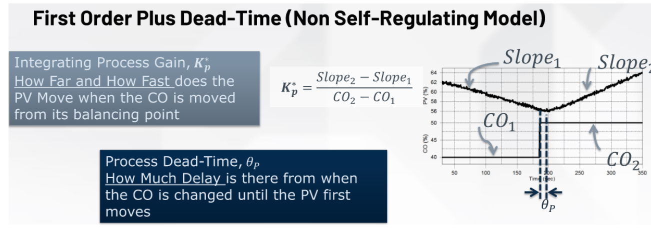


Figure 7
(Rice, 2024)

The fourth step in the process is to calculate the PID parameters. Use the equations below for a self-regulating process.

- First compute, τ_C, the closed loop time constant
  - aggressive: τ_C is the larger of $0.1\,\tau_p$ or $0.8\,\theta_p$
  - moderate: τ_C is the larger of $1\,\tau_p$ or $8\,\theta_p$
  - conservative: τ_C is the larger of $10\,\tau_p$ or $80\,\theta_p$

	P	I	D
PI	$\frac{1}{K_P}\frac{\tau_p}{(\theta_p + \tau_C)}$	τ_p	
PID	$\frac{1}{K_P}\frac{(\tau_p + 0.5\theta_p)}{(\tau_C + 0.5\theta_p)}$	$\tau_p + 0.5\theta_p$	$\frac{(\tau_p\theta_p)}{(2\tau_p + \theta_p)}$

Figure 8
(Rice, 2024)

And the equations below for a non-self-regulating process.

- The Closed Loop Time Constant, τ_C, should be as large as possible but still fast enough to arrest or recover from a major disturbance.

	P	I
PI	$\frac{1}{K_P^*}\left(\frac{2\tau_C + \theta_p}{(\tau_C + \theta_p)^2}\right)$	$2\tau_C + \theta_p$

Figure 9
(Rice, 2024)

The fifth step in the process is to implement and test. Implement the PID parameters calculated in step four then make several setpoint changes and observe how the process reacts. If the reaction is acceptable move on to step six, if not adjust the PID parameters using table 2 as a guide.

The sixth and final step in the process is to document your results. This last step may seem optional but can be invaluable down the road when troubleshooting the PID loop or when further changes are needed to tuning. The method of storing the information may be as simple as a logbook or as complex as version management software. At a minimum the record should show who made the change, what values were adjusted, when tuning occurred, and why.

Conclusion

Presented in this paper are three different PID tuning methods, each with their strengths and weaknesses. Tuning by feel can be quick but leads to inconsistent results. Zeigler-Nichols results in aggressive tuning that may be beneficial in some situations but may be a detriment in others. First-order plus dead time modeling offers a good balance of consistency and usability, for that reason it is recommended especially for more difficult to tune PID loops. All tuning methods suffer from one major drawback and that is that you're tuning the loop at a particular moment in time. Down the road when conditions change the tuning may no longer be ideal. Thus it is prudent to always keep an eye on the health of your PID loops.

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